647510 **BSc3MatC301x** **Seat No :________**

B.Sc. Semester - 3 (*CBCS*) Examination
Oct/Nov - 2025 [New Course]
Mathematics-M301(Core)

Time: 2:30 Hours **Marks:70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1(A) Attempt any TWO. (10)

1) Define a vector space. Prove or disprove : $(\mathbb{R}^2, \oplus, \odot)$ is a vector space over field $\mathbb{R}$, where operation $\oplus$ on $\mathbb{R}^2$ is defined by $(a,b) \oplus (c,d) = (a+c,\ b+d)$ and operation $\odot$ is defined by $\lambda \odot (a,\ b) = (\lambda\, a\ ,\ 0)$ for all $(a,\ b), (c,\ d) \in \mathbb{R}^2$ and $\lambda \in \mathbb{R}$.
2) If $A = \{v_1, v_2, v_3\}$ is a linearly dependent subset of $\mathbb{R}^3$ and $P \in M_3(\mathbb{R})$ be a matrix obtained by taking the vectors of A as respective column vectors of matrix P then prove that the determinant of P will be 0.
3) Prove that the span of $B = \mathbb{R}^3$ where $B = \{(1,2,1), (-1,1,0), (5,-1,2)\}$.

Que.1(B) Attempt any ONE. (04)

1) Prove that any superset of a linearly dependent set is always linearly dependent.
2) Prove or disprove: $W = \{(a,\ b,\ c) \in \mathbb{R}^3 \mid a^2+b^2+c^2 \geq 1\}$ is a subspace of $\mathbb{R}^3$.

Que.2(A) Attempt any TWO. (10)

1) State and prove dimension theorem for the sum of two subspaces of a vector space.
2) Let $W_1 = \{(x-y+z\ ,\ x+y-z\ ,\ y+z\ ,\ 2x-4y-4z) \mid x,y,z \in \mathbb{R}\}$ and $W_2 = \{(2x-4y+z\ ,\ x+y\ ,\ x-y\ ,\ z) \mid x,y,z \in \mathbb{R}\}$ be given subspaces of $\mathbb{R}^4$. Find the dimension of subspaces W_1 , W_2 , $W_1 \cap W_2$ and $W_1 + W_2$.
3) Prove that every linearly independent subset of a vector space can be extended to form its basis.

Que.2(B) Attempt any ONE. (04)

1) Does $\{(-1,1,0,0)\ ,\ (1,0,0,1)\} = sp\{(1,1,0,2)\ ,\ (1,2,0,3)\}$? Justify.
2) Define basis of a vector space. Extend set $\{(2,1,1,2),\ (1,0,0,1)\}$ to a basis of $\mathbb{R}^4$, if possible.

Que.3(A) Attempt any TWO. (10)

1) Define scalar point function. If f and g are differentiable scalar functions on the domain D of $\mathbb{R}^3$, then show that : (1) $\nabla(f^2) = 2f\ \nabla(f)$ and (2) $\nabla\left(\frac{f}{g}\right) = \frac{g\ \nabla f - f\ \nabla g}{g^2}$.
2) Define vector point function. If $\bar{f}$ and $\bar{g}$ are vector functions on the domain D whose first order partial derivatives exist, then show that $div\left(\bar{f} \times \bar{g}\right) = \bar{g} \cdot curl\ \bar{f} - \bar{f} \cdot curl\ \bar{g}$.
3) If $\bar{f}$ and $\bar{g}$ are irrotational functions on domain D then show that $\bar{f} \times \bar{g}$ is a solenoidal function.

Que.3(B) Attempt any ONE. (04)

1) Define gradient of a function. Find the unit normal on the surface $x^2y + 2xz = 4$ at point $(-2,1,3)$.
2) Prove that the function $H = e^{2x}(3\cos 2y - 5\sin 2y)$ satisfies the Laplace equation.

Que.4(A) Attempt any TWO. (10)

1) Evaluate $\iint_R x\cos(x+y)\,dx\,dy$ where region R is the triangle whose vertices are $(0,0)$, $(\pi,0)$ and (π,π) .

2) Evaluate $\iiint_R xyz\,dx\,dy\,dz$ where R is the region enclosed by the planes $x=0$, $y=0,\ z=0$ and $x+y+z=1$.

3) By using the concept of triple integral, prove that the volume of a sphere of radius a units is $\frac{4}{3}\pi a^3$ cubic units.

Que.4(B) Attempt any ONE. (04)

1) Find the Jacobian $|J|$ by change of variable from cartesian to spherical coordinate system.

2) Change the order of integration of $\int_0^1\int_{x^2}^{2-x} f(x,y)\,dy\,dx$ and hence evaluate it for $f(x,y)=xy$.

Que.5(A) Attempt any TWO. (10)

1) State and prove the Green's theorem.

2) State the Gauss divergence theorem and verify it for $\bar{F}=(x^2-y^2,\ 2xy,\ y^2+z)$, and S is the closed region obtained by planes $x=0,\ y=0,\ z=0,\ x=1$, $y=1$ and $z=1$.

3) Define beta function and prove that $\beta(m,n)=2\int_0^{\frac{\pi}{2}}\sin^{2m-1}\theta\,\cos^{2n-1}\theta\,d\theta$.

Que.5(B) Attempt any ONE. (04)

1) Using the Stoke's theorem, find $\oint_C \bar{V}\,ds$ for $\bar{V}=(x^2+y^2,\ -2xy,\ 0)$ where C is $x^2+y^2=1$.

2) Define gamma function and prove that $\Gamma(n)=\frac{1}{n}\int_0^\infty e^{-x^{1/n}}\,dx$.

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